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Simulating Nonlinearity in MEMS Resonators by a Charge Controlled Capacitor

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Abstract

In this paper, a novel circuit formulation allowing co-simulation of nonlinearity in MEMS resonators and interface circuits in a generic circuit simulation platform is reported for the first time. A charge controlled capacitor is used to model the nonlinear restoring force in the mechanical resonator. The concept has been applied to the case of a longitudinal beam resonator, illustrating the effect of feedthrough in the limit of nonlinear behavior characterized by features such as spring softening and hysteresis.

Keywords: MEMS; nonlinearity; equivalent circuit; resonator

1. Introduction

MEMS resonators have been exploited in both sensory and timing applications. Depending on the geometry, material and method of transduction, these vibratory devices exhibit nonlinear behavior in the limit of large displacement, thereby placing a limit on performance. As such, nonlinear analysis is of great importance particularly for high-Q MEMS resonators, where the displacement is largely constrained by nonlinearity. From this viewpoint, a simulation platform capable of modeling the electromechanical characteristics of a resonator under nonlinear conditions will be much welcomed. While elementary circuit blocks which map displacement to voltage have been proposed [1], this requires a custom software since the form of the circuit defers the conventional RLC used to represent a mass-damper-spring system. In this paper, we present an alternative convention where displacement is represented by the charge on a capacitor, which is further modified in a circuit simulation suite to describe nonlinear behavior. While the

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work here has been achieved in ADS, the concepts apply to all commonly used simulation platforms. The results here therefore demonstrate the possibility of performing a full system level simulation that includes both MEMS and interface circuits where nonlinearities can be accounted for.

2. Formulation for mechanical to electrical equivalence

The motion of a generic MEMS resonator can be modeled as a linear mass-damper-spring system:

$$m\ddot{x} + \gamma\dot{x} + f(x) = F(t) \tag{1}$$

where *m* is the mass, γ is the damping coefficient, *f(x)* and *F(t)* represent the restoring force of the spring and a sinusoidal driving force respectively. In the case of a linear system, $f(x) = kx$. While it has been shown that (1) can be modeled by a series RLC using the capacitor voltage to denote displacement [2], this formulation does not yield a unified agreement between all electrical and mechanical variables. In contrast, using charge (Q) to represent displacement provides such a unified agreement as summarized in the Table 1:

Table 1. Formulation for mechanical to electrical equivalence

	Constants			Variables	
Mechanical	Mass (m)	Damping coefficient (γ)	Spring constant (k)	Displacement (x)	Excitation force (F)
Electrical	Inductance (L)	Resistance (R)	Elastance ($1/C$)	Charge (Q)	Source voltage (U)

In the limit of nonlinearity, the Helmholtz-Duffing equation considers the higher order spring constants:

$$f(x) = k_1 x + k_2 x^2 + k_3 x^3 \tag{2}$$

where k_2 and k_3 are quadratic and cubic spring constants. Based on the formulation using Q to denote x, we can then construct a charge-controlled capacitor [3] to describe the polynomial function given by:

$$U = Q/C_1 + Q^2/C_2 + Q^3/C_3 \tag{3}$$

where C_2 and C_3 are second-order and third-order capacitance coefficients respectively. Since (3) has a similar form to (2), we may use the total charge Q on the capacitor to indicate the displacement of the resonator. Therefore, simulating the proposed RLC circuit will provide the exact value of displacement which is equivalent to the total charge stored on the capacitor.

Ultimately, this motion is captured as a current due to the modulation of the capacitive gap when a DC bias voltage is applied across the nominal gap (d). This effect can be modeled by a variable capacitor:

$$U = Q/C(x) = \int I_m \big/ \left(\varepsilon_0 A/(d-x)\right) \tag{4}$$

where *C(x)* is the capacitance of the transducer under different displacements and Q is the total charge stored on the transducer. Furthermore, the proposed variable capacitor also allows for the presence of high-order harmonics stemming from the feedthrough to be captured, which would otherwise be seen as merely amplitude changes at the fundamental tone as the excitation voltage is increased.

3. Case Study

The reported parameters for a nonlinear electrostatic longitudinal beam resonator by [4] were adopted in our model for validation. The resonator's significant quadratic mechanical spring constant and strong spring softening effect makes for an excellent case study to apply our circuit model, which is shown in Fig. 1. The lumped parameters of the longitudinal bulk mode beam resonator are listed in Table 2.

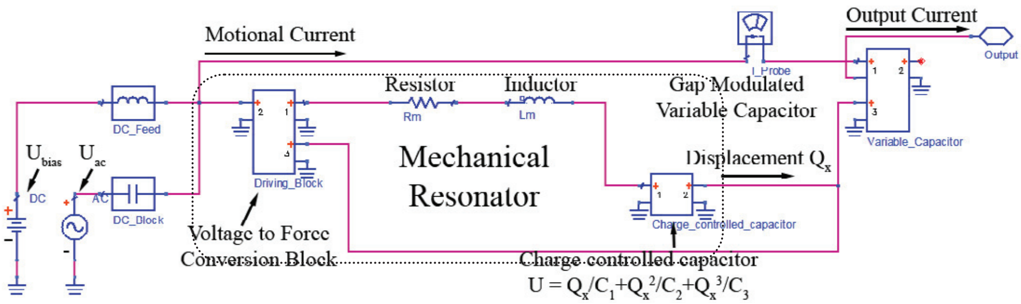


Fig. 1. Schematic of the equivalent circuit

Table 2. Lumped parameters of the longitudinal beam resonator of gap d = 1μm

Effective Mass (m) kg	Damping Coefficient (γ) N•s/m	Linear Spring Constant (k_1) N/m	Quadratic Spring Constant (k_2) N/m^2	Cubic Spring Constant (k_3) N/m^3	Transduction Capacitor (C_t) fF
16.8×10^{-12}	6.88×10^{-9}	91.4×10^{3}	-1.764×10^{9}	-4.23×10^{13}	0.708

The harmonic balance (HB) method was used to study the nonlinear features of the resonator in the frequency domain. Firstly, only electrostatic nonlinear effects were considered by setting the higher order spring constants to zero. Fig. 2(a) shows the simulated displacement for increasing AC input, illustrating skewing in the resonant peak towards lower frequencies due to spring softening. Inclusion of higher order spring constants is shown to intensify the spring softening effects when we compare Fig. 2(b) to Fig. 2(a); where the maximum frequency shift increases from 40Hz to 250Hz for the same AC excitation inputs. Moreover, hysteresis can be observed as the AC input excitation increases as highlighted by the red dotted lines. The resonator becomes more nonlinear and easier to reach the hysteresis regime by adding the mechanical nonlinearity.

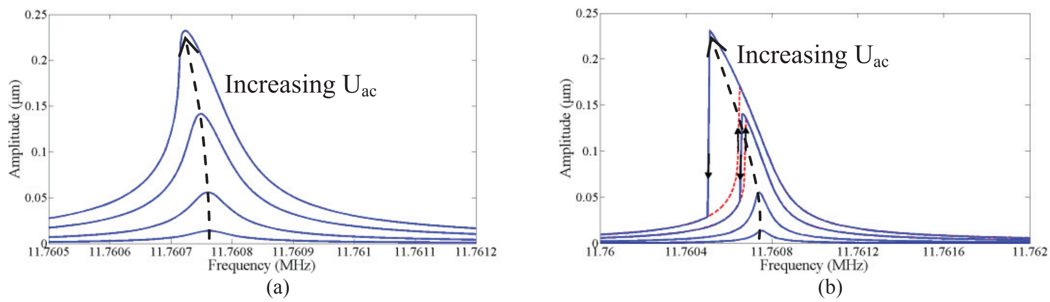
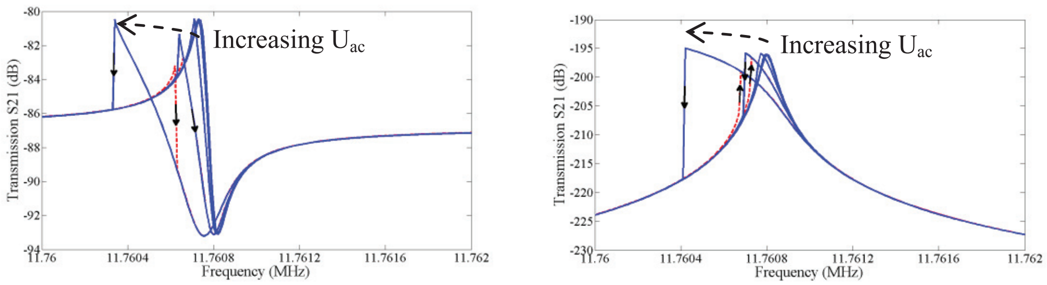


Fig. 2. Displacement with increasing AC input with higher-order spring constants (a) excluded; (b) included

Finally, we simulate the full electrical characteristics of the MEMS resonator where the output has both the motional as well as feedthrough currents as observed in actual measurements. Apart from the direct coupling through the capacitive transducer (i.e. variable capacitor), a parasitic coupling capacitor connecting the drive and sense ports was added to account for coupling through the substrate and bond wires. Fig. 3(a) shows the magnitude of the full electrical transmission characteristic (S21) that has been embedded in feedthrough with the output connected to a 50Ω termination load. The anti-resonance valley due to feedthrough can be clearly observed along with the increased skewing in the resonant peak towards lower frequencies as the AC excitation input is increased. Furthermore, the hysteresis becomes more clearly observable with the increase of the AC driving voltage. By re-routing the AC input to feed only into the mechanical resonator (variable capacitor thus only sees a DC bias), the S21 transmission curves containing only the motional current are obtained are shown in Fig. 3(b).



(a) (b)

Fig. 3. Magnitude of the S21 transmission with feedthrough (a) included; (b) excluded

4. Conclusion

We have used a charge controlled nonlinear capacitor model with higher order capacitance coefficients to capture the effects of both mechanical and electrostatic nonlinearity in MEMS resonators. A variable capacitor model was implemented to capture the modulating effect of the transducer gap. We show that the whole equivalent circuit can well predict the nonlinear effects for micro-resonators, demonstrating the possibility for system-level simulation of non-linear MEMS resonators together with interface circuits. In short, we demonstrate this capability for implementation in commonly used circuit simulation platforms.

Acknowledgement

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